HIGHER 2005 PAPER 2

ALL questions should be attempted.

1. Find $\int \dfrac{4x^3 - 1}{x^2}\, dx$, $x \neq 0$.
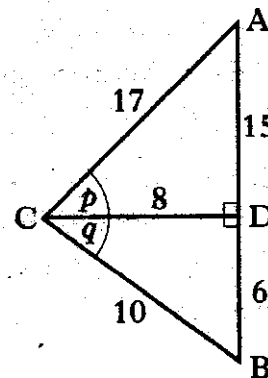
    4

2. Triangles ACD and BCD are right-angled at D with angles p and q and lengths as shown in the diagram.

    (a) Show that the exact value of $\sin(p + q)$ is $\dfrac{84}{85}$.

    4

    (b) Calculate the exact values of:

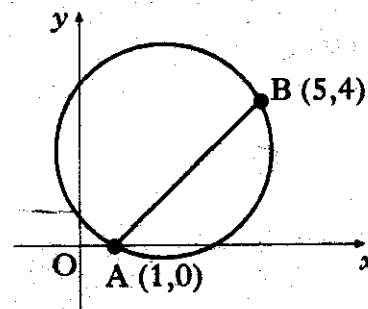   (i) $\cos(p + q)$;

    (ii) $\tan(p + q)$.

    3

3. (a) A chord joins the points A(1,0) and B(5,4) on the circle as shown in the diagram.
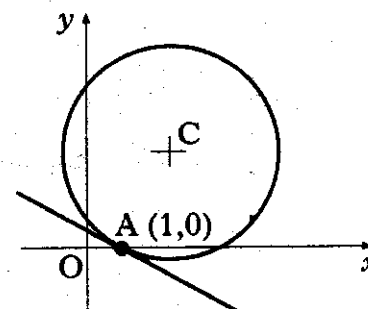
    Show that the equation of the perpendicular bisector of chord AB is $x + y = 5$.

    4

    (b) The point C is the centre of this circle. The tangent at the point A on the circle has equation $x + 3y = 1$.

    Find the equation of the radius CA.

    4

    (c) (i) Determine the coordinates of the point C.
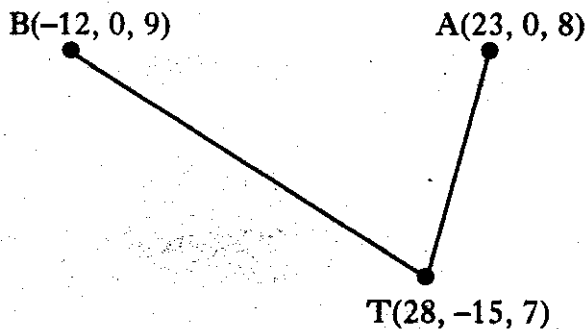
    (ii) Find the equation of the circle.

    4

4. The sketch shows the positions of Andrew(A), Bob(B) and Tracy(T) on three hill-tops.

    Relative to a suitable origin, the coordinates (in hundreds of metres) of the three people are A(23, 0, 8), B(−12, 0, 9) and T(28, −15, 7).

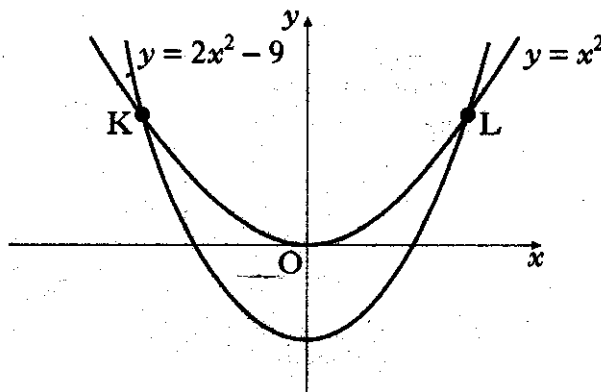   In the dark, Andrew and Bob locate Tracy using heat-seeking beams.

    B(−12, 0, 9)   A(23, 0, 8)

    T(28, −15, 7)

    (a) Express the vectors $\overrightarrow{TA}$ and $\overrightarrow{TB}$ in component form.   **2**

    (b) Calculate the angle between these two beams.   **5**

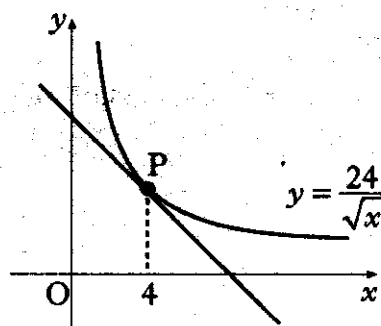5. The curves with equations $y = x^2$ and $y = 2x^2 − 9$ intersect at K and L as shown.

    Calculate the area enclosed between the curves.   **8**

    $y = 2x^2 − 9$ $y = x^2$

    K   L

6. The diagram shows the graph of $y = \dfrac{24}{\sqrt{x}}$, $x > 0$.

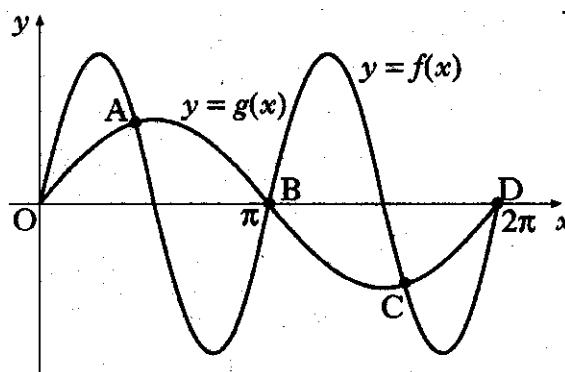   Find the equation of the tangent at P, where $x = 4$.   **6**

    P

    $y = \dfrac{24}{\sqrt{x}}$

7. Solve the equation $\log_4(5 − x) − \log_4(3 − x) = 2$, $x < 3$.   **4**

8. Two functions, f and g, are defined by $f(x) = k\sin 2x$ and $g(x) = \sin x$ where $k > 1$.

The diagram shows the graphs of $y = f(x)$ and $y = g(x)$ intersecting at O, A, B, C and D.

Show that, at A and C, $\cos x = \dfrac{1}{2k}$.

5

9. The value V (in £ million) of a cruise ship t years after launch is given by the formula $V = 252e^{-0.06335t}$.

   (*a*) What was its value when launched? **1**

   (*b*) The owners decide to sell the ship once its value falls below £20 million. After how many years will it be sold? **4**

10. Vectors a and c are represented by two sides of an equilateral triangle with sides of length 3 units, as shown in the diagram.

Vector b is 2 units long and b is perpendicular to both a and c.

Evaluate the scalar product $a.(a + b + c)$.
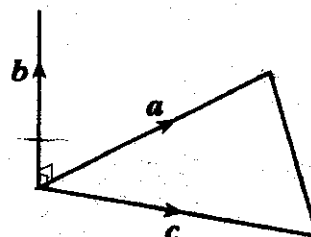
4

11. (*a*) Show that $x = -1$ is a solution of the cubic equation $x^3 + px^2 + px + 1 = 0$. **1**

   (*b*) Hence find the range of values of p for which all the roots of the cubic equation are real. **7**

[END OF QUESTION PAPER]