

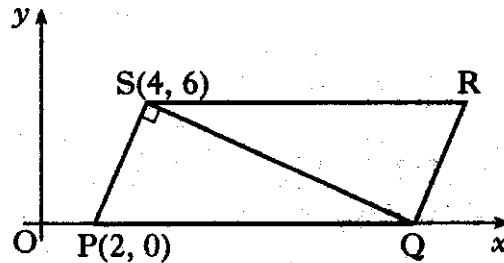
# HIGHER 2006 PAPER 2

ALL questions should be attempted.

Marks

1. PQRS is a parallelogram. P is the point (2, 0), S is (4, 6) and Q lies on the x-axis, as shown.

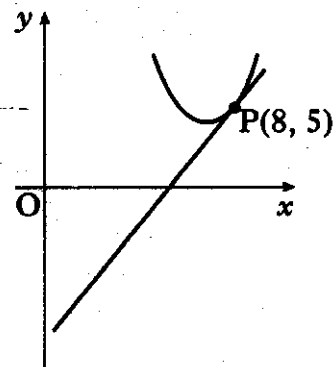
The diagonal QS is perpendicular to the side PS.



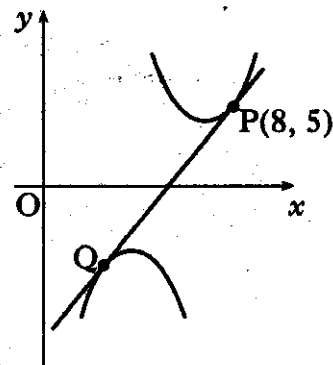
- (a) Show that the equation of QS is  $x + 3y = 22$ . 4
- (b) Hence find the coordinates of Q and R. 2
2. Find the value of  $k$  such that the equation  $kx^2 + kx + 6 = 0$ ,  $k \neq 0$ , has equal roots. 4

3. The parabola with equation  $y = x^2 - 14x + 53$  has a tangent at the point P(8, 5).

(a) Find the equation of this tangent.



(b) Show that the tangent found in (a) is also a tangent to the parabola with equation  $y = -x^2 + 10x - 27$  and find the coordinates of the point of contact Q.



4. The circles with equations  $(x - 3)^2 + (y - 4)^2 = 25$  and  $x^2 + y^2 - kx - 8y - 2k = 0$  have the same centre.

Determine the radius of the larger circle.

5

5. The curve  $y = f(x)$  is such that  $\frac{dy}{dx} = 4x - 6x^2$ . The curve passes through the point  $(-1, 9)$ . Express  $y$  in terms of  $x$ .

4

6. P is the point  $(-1, 2, -1)$  and Q is  $(3, 2, -4)$ .

(a) Write down  $\vec{PQ}$  in component form.

1

(b) Calculate the length of  $\vec{PQ}$ .

1

(c) Find the components of a unit vector which is parallel to  $\vec{PQ}$ .

1

7. The diagram shows the graph of a function  $y = f(x)$ .

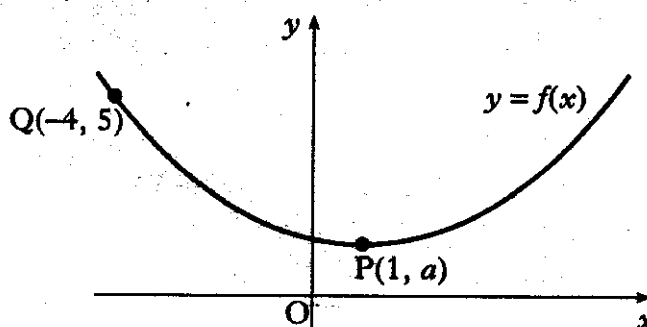
Copy the diagram and on it sketch the graphs of:

(a)  $y = f(x - 4)$ ;

2

(b)  $y = 2 + f(x - 4)$ .

2

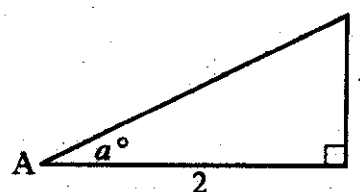


8. The diagram shows a right-angled triangle with height 1 unit, base 2 units and an angle of  $a^\circ$  at A.

(a) Find the exact values of:

(i)  $\sin a^\circ$ ;

(ii)  $\sin 2a^\circ$ .



4

(b) By expressing  $\sin 3a^\circ$  as  $\sin(2a + a)^\circ$ , find the exact value of  $\sin 3a^\circ$ .

4

9. If  $y = \frac{1}{x^3} - \cos 2x$ ,  $x \neq 0$ , find  $\frac{dy}{dx}$ .

4

10. A curve has equation  $y = 7\sin x - 24\cos x$ .

(a) Express  $7\sin x - 24\cos x$  in the form  $k\sin(x - a)$  where  $k > 0$  and  $0 \leq a \leq \frac{\pi}{2}$ .

4

(b) Hence find, in the interval  $0 \leq x \leq \pi$ , the  $x$ -coordinate of the point on the curve where the gradient is 1.

3

11. It is claimed that a wheel is made from wood which is over 1000 years old.

To test this claim, carbon dating is used.

The formula  $A(t) = A_0 e^{-0.000124t}$  is used to determine the age of the wood, where  $A_0$  is the amount of carbon in any living tree,  $A(t)$  is the amount of carbon in the wood being dated and  $t$  is the age of the wood in years.

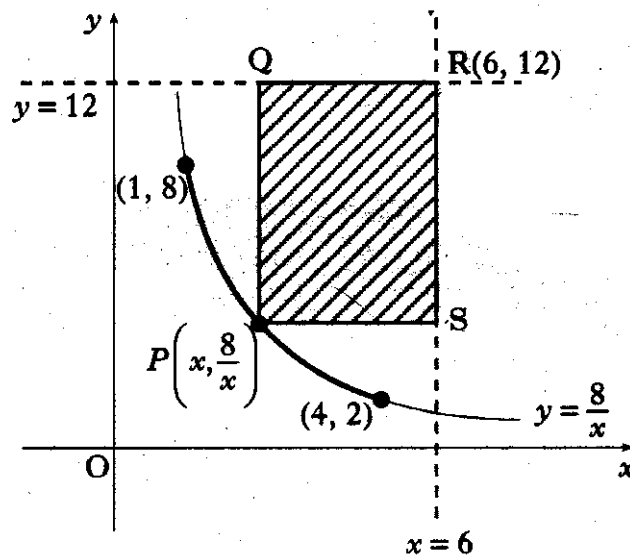
For the wheel it was found that  $A(t)$  was 88% of the amount of carbon in a living tree.

Is the claim true?

5

12. PQRS is a rectangle formed according to the following conditions:

- it is bounded by the lines  $x = 6$  and  $y = 12$
- P lies on the curve with equation  $y = \frac{8}{x}$  between  $(1, 8)$  and  $(4, 2)$
- R is the point  $(6, 12)$ .



- (a) (i) Express the lengths of PS and RS in terms of  $x$ , the  $x$ -coordinate of P.  
 (ii) Hence show that the area,  $A$  square units, of PQRS is given by

$$A = 80 - 12x - \frac{48}{x}.$$

3

- (b) Find the greatest and least possible values of  $A$  and the corresponding values of  $x$  for which they occur.

8

[END OF QUESTION PAPER]