Solving Trigonometric Equations

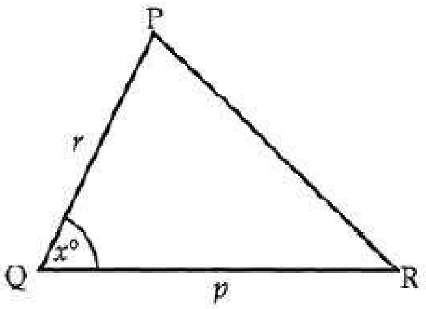
1. Solve the equation $3\cos 2x^\circ + \cos x^\circ = -1$ in the interval $0 \leq x \leq 360$. **5**

2. The diagram shows an isosceles triangle PQR in which PR = QR and angle PQR = x°.

(a) Show that $\frac{\sin x^\circ}{p} = \frac{\sin 2x^\circ}{r}$. **(3)**

(b) (i) State the value of x° when $p = r$.

(ii) Using the fact that $p = r$, solve the equation in (a) above, to justify your stated value of x°. **(5)**



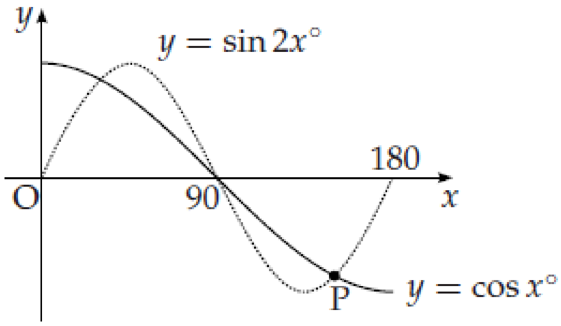
3. If $f(a) = 6\sin^2 a - \cos a$, express $f(a)$ in the form $p\cos^2 a + q\cos a + r$.

Hence solve, correct to three decimal places, the equation $6\sin^2 a - \cos a = 5$ for $0 \leq a \leq \pi$. **4**

4. (a) Solve the equation $\sin 2x^\circ - \cos x^\circ = 0$ in the interval $0 \leq x \leq 180$. **4**

(b) The diagram shows parts of two trigonometric graphs, $y = \sin 2x^\circ$ and $y = \cos x^\circ$.

Use your solutions in (a) to write down the coordinates of the point P. **1**



5. Functions f and g are defined on suitable domains by $f(x) = \sin(x^\circ)$ and $g(x) = 2x$.

(a) Find expressions for:

(i) $f(g(x))$;

(ii) $g(f(x))$. **2**

(b) Solve $2f(g(x)) = g(f(x))$ for $0 \leq x \leq 360$. **5**

6. Solve $2\sin 3x^\circ - 1 = 0$ for $0 \leq x \leq 180$. **4**

7. Solve the equation $2\cos^2 x = \frac{1}{2}$, for $0 \leq x \leq \pi$. **3**

8. Solve the equation $2\sin\left(2x - \frac{\pi}{6}\right) = 1$, $0 \leq x < 2\pi$. **4**

9. $f(x) = 2\cos x^\circ + 3\sin x^\circ$.

(*a*) Express $f(x)$ in the form $k\cos(x-\alpha)^\circ$ where $k>0$ and $0 \leq \alpha < 360$. **(4)**

(*b*) Hence solve algebraically $f(x) = 0.5$ for $0 \leq x < 360$. **(3)**

10. The function f is defined by $f(x) = 2\cos x^\circ - 3\sin x^\circ$.

(*a*) Show that $f(x)$ can be expressed in the form $f(x) = k\cos(x+\alpha)^\circ$ where $k>0$ and $0 \leq \alpha < 360$, and determine the values of k and α. **(4)**

(*b*) Hence find the maximum and minimum values of $f(x)$ and the values of x at which they occur, where x lies in the interval $0 \leq x < 360$. **(4)**

(*c*) Write down the minimum value of $(f(x))^2$. **(1)**

11. Solve the equation $2\sin x^\circ - 3\cos x^\circ = 2{\cdot}5$ in the interval $0 \leq x < 360$. **8**