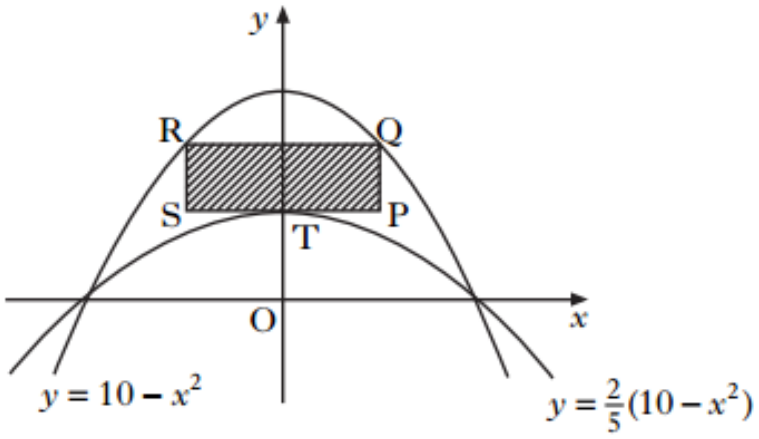
The parabolas with equations $y = 10 - x^2$ and $y = \frac{2}{5}(10 - x^2)$ are shown in the diagram below.



A rectangle PQRS is placed between the two parabolas as shown, so that:

- Q and R lie on the upper parabola;
- RQ and SP are parallel to the x-axis;
- T, the turning point of the lower parabola, lies on SP.

(*a*) (i) If TP = x units, find an expression for the length of PQ.

(ii) Hence show that the area, A, of rectangle PQRS is given by

$$A(x) = 12x - 2x^3.$$ **3**

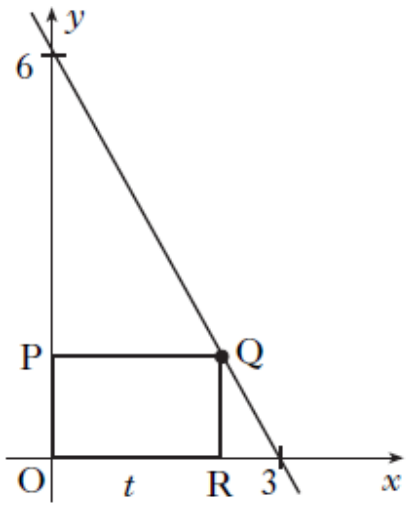
(*b*) Find the maximum area of this rectangle. **6**

In the diagram, Q lies on the line joining (0, 6) and (3, 0).

OPQR is a rectangle, where P and R lie on the axes and OR = t.

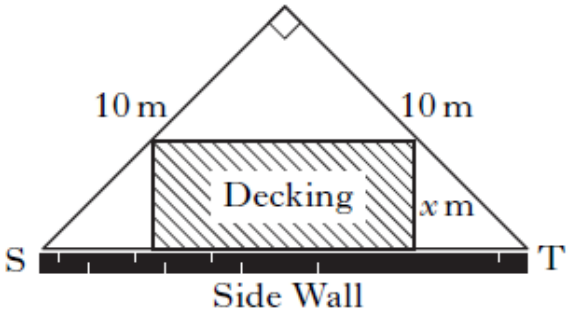
(*a*) Show that QR = $6 - 2t$. **3**

(*b*) Find the coordinates of Q for which the rectangle has a maximum area. **6**



A householder has a garden in the shape of a right-angled isosceles triangle.

It is intended to put down a section of rectangular wooden decking at the side of the house, as shown in the diagram.



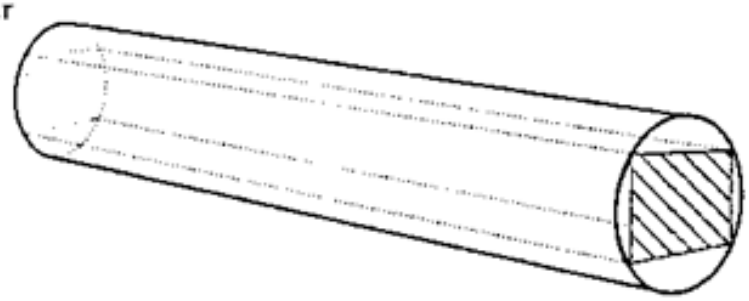
(*a*) (i) Find the exact value of ST.

(ii) Given that the breadth of the decking is x metres, show that the area of the decking, A square metres, is given by

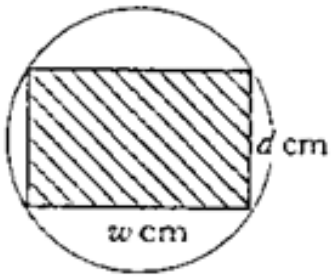
$$A = \left(10\sqrt{2}\right)x - 2x^2.$$ **3**

(*b*) Find the dimensions of the decking which maximises its area. **5**

A rectangular beam is to be cut from a cylindrical log of diameter 20 cm.



The diagram shows a cross-section of the log and beam where the beam has a breadth of w cm and a depth of d cm.



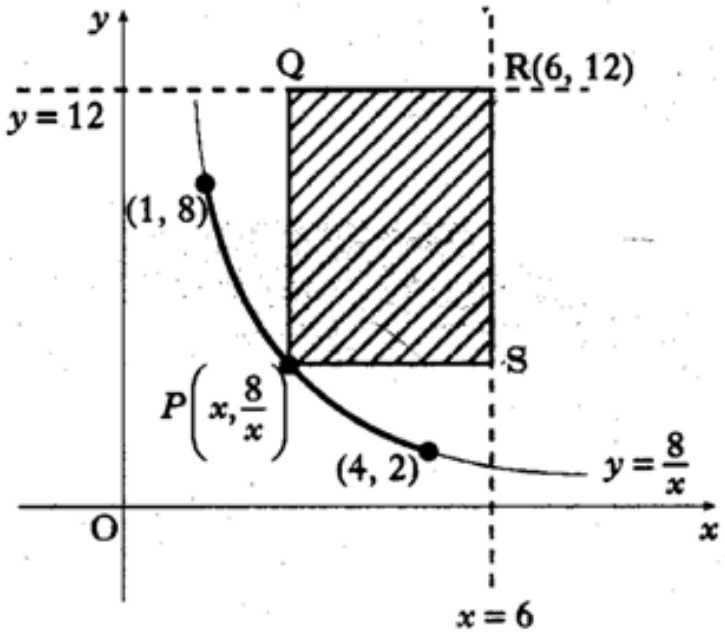
The strength S of the beam is given by

$S = 1{\cdot}7\, w\, (400 - w^2)$.

Find the dimensions of the beam for maximum strength. **5**

PQRS is a rectangle formed according to the following conditions:

- it is bounded by the lines $x = 6$ and $y = 12$
- P lies on the curve with equation $y = \frac{8}{x}$ between (1, 8) and (4, 2)
- R is the point (6, 12).



(*a*) (i) Express the lengths of PS and RS in terms of x, the x-coordinate of P.

(ii) Hence show that the area, A square units, of PQRS is given by

$$A = 80 - 12x - \frac{48}{x}.$$ **3**

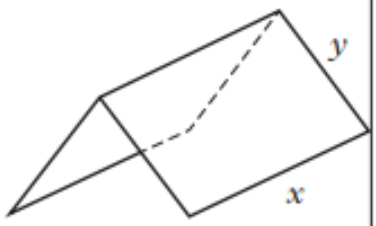
(*b*) Find the greatest and least possible values of A and the corresponding values of x for which they occur. **8**

A manufacturer is asked to design an open-ended shelter, as shown, subject to the following conditions.

Condition 1

The frame of a shelter is to be made of rods of two different lengths:

- x metres for top and bottom edges;
- y metres for each sloping edge.



Condition 2

The frame is to be covered by a rectangular sheet of material.

The total area of the sheet is $24\,m^2$.

(*a*) Show that the total length, L metres, of the rods used in a shelter is given by

$$L = 3x + \frac{48}{x}.$$

3

(*b*) These rods cost £8·25 per metre.

To minimise production costs, the total length of rods used for a frame should be as small as possible.

(i) Find the value of x for which L is a minimum.

(ii) Calculate the minimum cost of a frame.

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