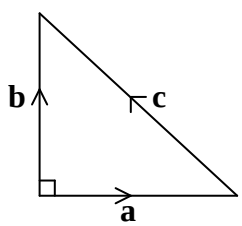
Section 2: Vectors

1. ABCD is a quadrilateral with vertices A(4,-1,3), B(8,3,-1), C(0,4,4) and D(-4,0,8).
M is the mid-point of AB.
   (a) Find the coordinates of M.
   (b) Find the coordinates of T, which divides CM in the ratio 2:1.
   (c) Show that B, T and D are collinear, and find the ratio in which T divides BD.

2. A(4,4,10) , B(-2,-4,12) and C(-8,0,10) are the vertices of a right-angled triangle.
Which angle of triangle ABC is the right angle?

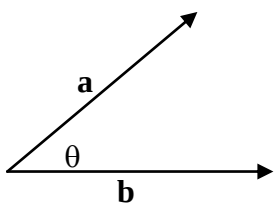
3. $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$. Show that **a** and **b** are perpendicular.

4. 

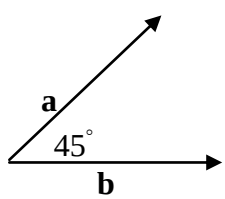
This triangle is right-angled and isosceles.
The equal sides have length 2 units.
Find the value of $\mathbf{b}.(\mathbf{a}+\mathbf{b}+\mathbf{c})$.

5. In the diagram below, $\mathbf{a} = \begin{pmatrix} 4 \\ -4 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}$.

Show that $\cos\theta = -\frac{7}{9}$.



6. A, B, C, and D have coordinates (-1,3,1), (1,6,7), (0,2,5) and (1,4,10), respectively.
   (a) Find the components of $\overrightarrow{\mathbf{AB}}$ and $\overrightarrow{\mathbf{CD}}$.
   (b) The vector $\begin{pmatrix} p \\ q \\ 1 \end{pmatrix}$ is perpendicular to both $\overrightarrow{\mathbf{AB}}$ and $\overrightarrow{\mathbf{CD}}$. Find p and q.
   (c) Find a vector of unit length parallel to $\overrightarrow{\mathbf{AB}}$.

7. 

$|\mathbf{a}| = 3$ and $|\mathbf{b}| = 2\sqrt{2}$.

   (a) Evaluate (i) $\mathbf{a}.\mathbf{a}$ (ii) $\mathbf{b}.\mathbf{b}$ (iii) $\mathbf{a}.\mathbf{b}$
   (b) If $\mathbf{p} = 2\mathbf{a} + 3\mathbf{b}$, evaluate $\mathbf{p}.\mathbf{p}$ and hence write down $|\mathbf{p}|$.