

## recurrence relations

- [SQA] 1. The sum of £1000 is placed in an investment account on January 1st and, thereafter, £100 is placed in the account on the first day of each month.
- Interest at the rate of 0.5% per month is credited to the account on the last day of each month.
  - This interest is calculated on the amount in the account on the first day of the month.
- (a) How much is in the account on June 30th? (4)
- (b) On what date does the account first exceed £2000? (2)
- (c) Find a recurrence relation which describes the amount in the account, explaining your notation carefully. (3)
- [SQA] 2. A gardener feeds her trees weekly with “Bioforce, the wonder plant food”. It is known that in a week the amount of plant food in the tree falls by about 25%.
- (a) The trees contain no Bioforce initially and the gardener applies 1g of Bioforce to each tree every Saturday. Bioforce is only effective when there is continuously more than 2g of it in the tree. Calculate how many weekly feeds will be necessary before the Bioforce becomes effective. (3)
- (b) (i) Write down a recurrence relation for the amount of plant food in the tree immediately after feeding. (1)
- (ii) If the level of Bioforce in the tree exceeds 5g, it will cause leaf burn. Is it safe to continue feeding the trees at this rate indefinitely? (4)

- [SQA] 3. On the first day of March, a bank loans a man £2500 at a fixed rate of interest of 1.5% per month. This interest is added on the last day of each month and is calculated on the amount due on the first day of the month. He agrees to make repayments on the first day of each subsequent month. Each repayment is £300 except for the smaller final amount which will pay off the loan.
- (a) The amount that he owes at the start of each month is taken to be the amount still owing just after the monthly repayment has been made.  
Let  $u_n$  and  $u_{n+1}$  represent the amounts that he owes at the start of two successive months. Write down a recurrence relation involving  $u_{n+1}$  and  $u_n$ . 2
- (b) Find the date and the amount of the final payment. 4
- [SQA] 4. Two sequences are generated by the recurrence relations  $u_{n+1} = au_n + 10$  and  $v_{n+1} = a^2v_n + 16$ .  
The two sequences approach the same limit as  $n \rightarrow \infty$ .  
Determine the value of  $a$  and evaluate the limit. 5
- [SQA] 5. A man decides to plant a number of fast-growing trees as a boundary between his property and the property of his next door neighbour. He has been warned, however, by the local garden centre that, during any year, the trees are expected to increase in height by 0.5 metres. In response to this warning he decides to trim 20% off the height of the trees at the start of any year.
- (a) If he adopts the "20% pruning policy", to what height will he expect the trees to grow in the long run? 3
- (b) His neighbour is concerned that the trees are growing at an alarming rate and wants assurances that the trees will grow no taller than 2 metres. What is the minimum percentage that the trees will need to be trimmed each year so as to meet this condition. 3

[END OF QUESTIONS]